

MOND cosmology from holographic principle

Hongsheng Zhang* and Xin-Zhou Li †

*Shanghai United Center for Astrophysics (SUCA),
Shanghai Normal University, 100 Guilin Road, Shanghai 200234, China*

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We derive the MOND cosmology which is uniquely corresponding to the original MOND in galaxies via holographic approach of gravity. It inherits the key merit of MOND, that is, it reduces the byronic matter and mysterious non-byronic dark matter (dark matter for short) in the standard cosmology into byronic matter only. For the first time we derive the critical parameter in MOND, i.e., the transition acceleration a_c on cosmological scale. We thus solve the long-standing coincidence problem $a_c \sim cH_0$. More interestingly, a term like age-graphic dark energy emerges naturally. In the frame of this MOND cosmology, we only need byronic matter to describe both dark matter and dark energy in standard cosmology.

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I. INTRODUCTION

Dark matter was believed to be responsible for the flat rotation curves of galaxies for many years [1]. Further study shows that dark matter shares almost the same distribution in all the observed galaxies, which is independent on the type and compositions of the galaxies. A nice analogy the situation is the equivalence principle, which says that all the objects share the same acceleration, which is independent on the stuff of the objects. More importantly, we never directly observe the dark matter particles. We only have some indirect evidence from its gravitational effect. Thus, a natural idea is that the surplus gravitational effect is the property of the gravity itself, not some mysterious invisible matter. It was suggested that Newtonian dynamics is modified through modification in the kinematic a as effective acceleration $a_N = a\mu(a/a_c)$, where $\mu = 1$ for the usual-valued accelerations but $\mu = a/a_c$ when acceleration is much smaller than a critical acceleration a_c [2]. a_c is a new scale with the same dimension of a . For recent reviews and more references about this modified Newtonian dynamics (MOND), see [3].

MOND gives excellent explanations of the rotation curves of very different types of galaxies with the same acceleration transition parameter a_c . This success is not surprising since MOND is designed to do so. Moreover, MOND can explain several other independent observations. For instance, Tully-Fisher relation [4] is almost axiomatic in MOND paradigm, but needs schemed distribution in dark matter senecio. We know MOND must be improved though it has splendid achievement at the galaxy scale, since it could not describe the dynamics of the universe. A systematic study of spiral galaxies pins a_c to be: $a_c = 1.2 \times 10^{-8} \text{cm sec}^{-2} = 0.176cH_0$, where c is the luminous velocity, and H_0 denotes the present

Hubble parameter. This is a remarkable indication that MOND should be generalized to cosmic scale: a_c is at the scale of the present Hubble parameter $a_c \sim cH_0$, which has been pointed out several years before [5].

A number of relativistic MOND theories appear recently. A leading relativistic MOND is tensor-vector-scalar (TeVeS) formulation [6]. Generally there are two scalars, one vector, and one tensor together with one arbitrary function in TeVeS, which makes the theory become very complicated. The TeVeS frame involves an arbitrary function in addition to many parameters as well as auxiliary fields, thus it is difficult to make a testable prediction in cosmology, though we can get Λ CDM-like cosmology by fixing some special parameter [7]. Surely, we should reduce freedoms in this model, the best is a theory which is uniquely corresponding to the original MOND without additional fields and arbitrary function. We shall try to find such a theory by holographic approach of gravity (entropic force, EF). Further, we shall show that such an extrapolation of MOND to the universe is well consistent with observations.

An interesting derivation of Newtonian law and Einstein field equation is considered in [8], in which space is emergent in holographic approach, and gravity emerges when the entropy associated with the position of an object changes. In cosmology, the Friedmann equation also can be derived by EF method [9, 10]. EF is a general paradigm which has been used to modified gravity. For example, we can change the equipartition law with Debye distribution to get a cosmological model in which the universe accelerate without invoking dark energy [11]. In this letter, we prove that there exists a unique cosmology as extension of the original MOND at galaxy scale. We shall show that it inherits the critical property of MOND (replacing dark matter), and bears with an innate term to accelerate the late universe, which has not been mined in MOND before.

This paper is organized as follows: In the next section, we derive the Friedmann equation of MOND cosmology by applying EF method to an FRW universe, based on which we explore the primary properties of MOND cos-

*Electronic address: hongsheng@shnu.edu.cn

†Electronic address: kyhz@shnu.edu.cn

mology. In section III, we conclude this paper.

II. MOND COSMOLOGY

It is not a completely new idea that gravity acts as a mean field something like water in hydrodynamics rather than a fundamental interaction in nature [12]. It is shown that Einstein field equation can be regarded as state equation in view of thermodynamics [13]. Recently, it is found that Einstein field equation is equal to first law of thermodynamics in spherically static symmetric space-time [14]. It is especially important that one studies the relation between thermodynamics and gravity can not be obtained from the first principle [15]. Now we develop the method in [10] to deduce the Friedmann equation in MOND cosmology.

We start from an homogeneous and isotropic 4-dimensional universe, which is described by an FRW metric,

$$ds^2 = -dt^2 + R^2 d\Omega_3^2, \quad (1)$$

where R denotes the scale factor, and Ω_3 represents a 3-Euclidean space, 3-sphere, or 3-pseudosphere. Following [8, 10], we consider a comoving spatial spherical 3-volume V with a 2-boundary S as the holographic screen of V , which loads complete information in V . We introduce the prime assumption in EF approach, which says the number of bits associated to the screen S is proportional to its area,

$$N = \frac{A}{G}, \quad (2)$$

where A denotes the area of S , and as usual G is the Newton gravitational constant. Warranted by the equipartition law of energy the total energy of the screen E reads,

$$E = \frac{1}{2} N k_B T, \quad (3)$$

where T is the temperature of the comoving observer sensed by static observer (whose physical coordinates are constants) in this dynamical FRW universe, k_B is the Boltzman's constant. Similar to the arguments in [8], the mass M enclosed in V equals to E ,

$$E = M. \quad (4)$$

The temperature of the comoving observer with acceleration a relative to observers with constant physical coordinates can be obtained from Unruh's reasoning [16],

$$T = \frac{a}{2\pi k_B}. \quad (5)$$

a can be derived directly [10],

$$a = -\ddot{R}r, \quad (6)$$

where r is the radial comoving coordinate of the comoving observer, hence its physical coordinate r_{ph} is $r_{ph} = Rr$. Here $r \in (0, 1)$ from the Local Group to the boundary of the observed universe.

Up to now, everything is standard. Then we introduce the essential formula in MOND: the modified acceleration. An object with acceleration a is treated as an object with an effective acceleration a_N

$$a_N = a\mu(a/a_c), \quad (7)$$

in MOND paradigm, where μ is a smooth function with properties that $\mu = 1$ when $a/a_c \gg 1$ and $\mu = a/a_c$ when $a/a_c \ll 1$. Then correspondingly, we use a_N to replace a in the Unruh formula (5).

Now we consider a matter source in perfect fluid form,

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \quad (8)$$

in which ρ and p denote the density and pressure measured by comoving observers. The Bianchi identity requires,

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (9)$$

where H stands for Hubble parameter. We calculate M enclosed in the boundary S for the energy-momentum (8). The mass which yields acceleration of the comoving observer is active (Tolman-Komar) mass,

$$M = 2 \int_V dV \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} \right) u^\mu u^\nu. \quad (10)$$

It is straitforward to obtain the acceleration by associating the above equations (2)-(5), and (10),

$$\frac{\ddot{R}}{R} \mu = -\frac{4\pi G}{3}(\rho + 3p). \quad (11)$$

It is clear that it just degenerates to the familiar form when $\mu = 1$ ($a/a_c \gg 1$). It deserves to note that there is an alternative explanation of modified acceleration (7) in MOND, that is, modified inertia rather than modified acceleration, which says that the inertia of an object reads [3],

$$m_N = m\mu. \quad (12)$$

Under this situation the Unruh temperature becomes,

$$T = \frac{a\mu}{2\pi k_B}. \quad (13)$$

At the same time, the active mass formulation (10) holds. It is easy to confirm that we get the same acceleration equation as (11) in this alternative explanation of MOND. From (11) and (9), we derive the Friedmann equation in MOND,

$$H^2 + \frac{k}{R^2} = \frac{8\pi G}{3\mu} \rho + \frac{1}{R^2 \mu} \int dt \dot{R}^2 \dot{\mu}, \quad (14)$$

where $k = 1, 0, -1$, depending on the spatial curvature. The first term of the RHS of (14) denotes the effects of byronic matter (the total dust matter in standard model), and the second term indicates the cosmic acceleration (can happen in the early universe or late universe).

In the present letter, we concentrate on the late time universe in MOND cosmology. To explain the observed cosmic evolution, we introduce the concept “virtual dark matter” and “virtual dark energy” in MOND cosmology [17], since almost all observations are fitted in Λ CDM, or its slight variation Scalar-CDM frame. The generic Friedmann equation in the 4-dimensional general relativity can be written as

$$H^2 + \frac{k}{R^2} = \frac{8\pi G}{3}(\rho_{by} + \rho_{dm} + \rho_{de}), \quad (15)$$

where the first, second, and third terms of RHS in the above equation represent the byronic matter, dark matter, and dark energy respectively. Comparing (15) with (14), we derive the virtual dark matter and dark energy in MOND cosmology,

$$\rho_{dm} = \left(\frac{1}{\mu} - 1\right)\rho, \quad (16)$$

and

$$\rho_{de} = \frac{3}{8\pi G} \left(\frac{1}{R^2 \mu} \int dt \dot{R}^2 \dot{\mu} \right). \quad (17)$$

Now it is easy to derive μ_0 by cosmological observations. The subscript 0 labels the present value of a quantity. Introducing a dimensionless parameter arising naturally in Λ CDM

$$\eta = \frac{\rho_{by}}{\rho_{dm} + \rho_{by}}, \quad (18)$$

we immediately obtain from (16)

$$\mu = \eta. \quad (19)$$

In MOND μ is not a certain function except the qualitative property that $\mu = 1$ when $a/a_c \gg 1$ and $\mu = a/a_c$ when $a/a_c \ll 1$. However, it is faithful that the main implications for galaxies caused by MOND do not depend on the specific form of μ . Here we adopt a fairly general form,

$$\mu = \frac{a/a_c}{[1 + (a/a_c)^\alpha]^{\frac{1}{\alpha}}}, \quad (20)$$

where α is a positive constant. Solving the above equation, we derive a/a_c ,

$$\frac{a}{a_c} = \frac{\eta}{(1 - \eta)^\alpha}. \quad (21)$$

According to the final result from WMAP+BAO+H0 Mean we have $\eta = 1/6$ and $H_0 = 70.2 \text{ kms}^{-1} \text{ Mpc}^{-1}$ [18]. From (6),

$$\frac{a_0}{R_0} = -\frac{\ddot{R}_0 r}{R_0}. \quad (22)$$

Note that a_0 is the present value of a , which is different from a_c . We set the dimensionless comoving coordinate $r \in (0, 1)$, and hence the scale factor R has a dimension of length. R is a geometric parameter, which is shared by frames of MOND cosmology and Λ CDM. We calculate $\frac{a_0}{R_0}$ in spatially flat Λ CDM. First,

$$\frac{\ddot{R}_0 r}{R_0} = - \left[\frac{4\pi G}{3} (\rho_{\text{tot}} + 3p_{\text{tot}}) \right]_0 r, \quad (23)$$

$$H_0^2 = \left[\frac{8\pi G}{3} \rho_{\text{tot}} \right]_0, \quad (24)$$

where subscript tot denotes the total cosmic fluids. From (23) and (24), we get

$$\frac{\ddot{R}_0}{R_0} = - \left[\frac{H^2}{2} \frac{\rho_{\text{tot}} + 3p_{\text{tot}}}{\rho_{\text{tot}}} \right]_0, \quad (25)$$

in which we set $r = 1$. If we cancel the subscript 0 in the above equation, we obtain the general equation which is tenable at any cosmic epoch.

Then, from (21, 22) and (25), we obtain

$$a_c = -q_0 \frac{(1 - \eta^\alpha)^{\frac{1}{\alpha}}}{\eta} c H_0, \quad (26)$$

where q denotes the deceleration parameter and we have used $R_0 = H_0^{-1}$. In (26), we have restored all the dimensions by including the luminous velocity c . If we set $\alpha = 3/10$, and borrow the numerical values of H_0 , q_0 , and η from [18], we reach

$$a_c = 1.2 \times 10^{-8} \text{ cm sec}^{-2} = 0.176 c H_0, \quad (27)$$

which is well consistent with the value fitted by latest data of spiral galaxies [19]. Thus for the first time, we get the transition parameter a_c in MOND from cosmological observations.

Now we consider the virtual dark energy term (17). Cosmic acceleration is one of the most striking discoveries over last century [20]. MOND is suggested much earlier than the discovery of the cosmic acceleration. Of course, it is not designed to account for this acceleration. We shall show that (17) can accelerate the universe, therefore an accelerating universe can be treated as a prediction in a sense.

We study the behavior of the virtual dark energy in a power law universe $R \sim t^n$. Any universe in a small time segment can be viewed as a power law universe. When $a/a_c \ll 1$, we get

$$\rho_{de} = \frac{3}{8\pi G} \frac{n^2(n-2)}{3n-4} \frac{1}{t^2}, \quad (28)$$

and when $a/a_c \gg 1$,

$$\rho_{de} = 0. \quad (29)$$

So, ρ_{de} becomes important when $a/a_c \ll 1$, i.e., in the late universe. Several notes on the dark energy (17) are listed as follows:

1. The late behavior of dark energy in MOND cosmology (28) also can be realized by Károlyházy relation, which is an effect of quantum mechanics in general relativity [21]. This kind of dark energy is called agegraphic dark energy in literatures.

2. In the early time when $a/a_c \gg 1$, it disappears spontaneously such that it does not affect structure formation, though structure formation in MOND cosmology is still not clear.

3. There is only one new parameter a_c in this MOND cosmology compared to Λ CDM and it is no cosmological constant in MOND cosmology. Totally, it has the same freedoms as Λ CDM. Furthermore, there is really no free parameter in the dark energy term (17) (if we set a zero integration constant).

4. This term appears in an integration form, which implies the evolution of the universe at any epoch depends on a period of the history of the universe. This non-localization property is a result of holographic principle.

Now we make an preliminary numerical study in this model. As we have mentioned, any universe in a short time segment can be regarded as a power-law one. We use this point to estimate the present power n and then the present deceleration parameter. Through comparing with Λ CDM, we reach

$$\frac{8\pi G\rho_{de0}}{3H_0^2} = 0.725. \quad (30)$$

Next, from (28) we obtain $n = 2.407$, which describes an accelerating universe whose deceleration parameter is -0.5813 . The result perfectly matches recent observations. Note that we do not use the geometric information from the fitting results of Λ CDM, but only borrow the partition of dark energy density. More serious results about the parameters in MOND cosmology need a global fitting different observations.

III. CONCLUSION

MOND is a very successful senecio in galaxy scale to take the place of dark matter. We make a direct generalization for MOND to the cosmological scale via EF method. We study the primary properties of this MOND cosmology in this letter. We find that MOND cosmology inherits the merits of the original MOND, that is, it can generate enough dust in view of standard model with only byronic matter. Moreover, by means of the partition of dark matter and byronic matter in Λ CDM we get the critical parameter a_c in MOND on the cosmological scale for the first time. The other interesting property of MOND cosmology is that a dark energy term which can accelerate the universe emerges spontaneously. We make a preliminary investigation about this term and find that it perfectly consistent with observations. Several details about this model need to explore further.

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